

## Appendix

By solving with respect to  $b$  the first order condition for firm 2's profit maximization at the location stage, we obtain the following critical values

- $b = -a + \frac{3}{2}\sqrt{2(1-w^2)}$
- $b = -a - \frac{3}{2}\sqrt{2(1-w^2)}$
- $b = \rho$ , where  $\rho$  is a root of the polynomial:

$$4x^4 + 6x^3a + (-30w^2 + 30)x^2 + (-2a^3 + 21aw^2 - 21a)x - 2a^2 + 2a^2w^2 - 18 + 36w^2 - 18w^4 \quad (\text{A1})$$

Let us consider these solutions.

- Consider first the solution  $b = -a + \frac{3}{2}\sqrt{2(1-w^2)}$ . Given the reaction function of firm 1, we have to solve the following system in order to discuss the candidate optimal locations of firm 1 and firm 2:

$$\begin{aligned} a &= \frac{5}{4}b - \frac{1}{4}\sqrt{(9b^2 + 16(1-w^2))} \\ b &= -a + \frac{3}{2}\sqrt{2(1-w^2)} \end{aligned}$$

Again we have two solutions: the couple of locations  $a_1 = \frac{7}{6}\sqrt{2(1-w^2)}$  and  $b_1 = \frac{1}{3}\sqrt{2(1-w^2)}$ , and the couple  $a_2 = \frac{1}{3}\sqrt{2(1-w^2)}$ , and  $b_2 = \frac{7}{6}\sqrt{2(1-w^2)}$ . The first couple implies a value for  $b$  lower than  $a$ . This solution is therefore unacceptable. However, if eqts (14) and (15) were evaluated at the second couple, the marginal consumer would lie at  $\frac{3}{4}\sqrt{2(1-w^2)}$ .

Since the disequation  $\frac{3}{4}\sqrt{2(1-w^2)} > \frac{1-w}{2}$  is always satisfied, at this solution the firms' conjectures would not be fulfilled.

- It is easy to check that, if  $b = -\left(a + \frac{3}{2}\sqrt{2(1-w^2)}\right)$ , the values of  $b$  that solve the system of the two reaction functions are both smaller than  $a$ . This contradicts the assumption  $a < b$ .

We now consider the polynomial (A1). Its solutions obtained by substituting firm 1's optimal reply are:

$$\begin{aligned} x &= \sqrt{2(w^2 - 1)} \\ x &= -\frac{5}{18}\sqrt{6(1-w^2)} \\ x &= \frac{5}{18}\sqrt{6(1-w^2)} \end{aligned}$$

- We can immediately rule out the complex solution  $x = \sqrt{2(w^2 - 1)}$ .
- If  $b = -\frac{5}{18}\sqrt{6(1 - w^2)}$ , the optimal location of firm 1 is  $a = -\frac{29}{36}\sqrt{6(1 - w^2)}$ . In this case  $a < b$ , but both optimal solutions are negative and this contrasts again with the conjectures about the location of the marginal consumer.
- The last solution is indeed the only acceptable one. If  $b = \frac{5}{18}\sqrt{6(1 - w^2)}$ , then  $a = -\frac{1}{9}\sqrt{6(1 - w^2)}$ . Using these optimal locations into (14) and (15) we may verify that the marginal consumers is in the left external interval for  $w < \frac{1}{5}$ . Notice that this solution collapses to that obtained by Tabuchi and Thisse by setting  $w = 0$ .